UUCMS. No. | | | | | | | | | | | | |

B.M.S COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

V SEMESTER END EXAMINATION – JAN/FEB-2024

B.Sc – MATHEMATICS

MATHEMATICS-REAL ANALYSIS-II AND COMPLEX ANALYSIS

(NEP Scheme 2021-22 onwards F+R)

Course Code: MAT5DSC05 **QP Code:5024**

Duration: 2 ½ Hours **Max marks: 60**

Instructions: Answer all the sections.

SECTION-A

I. Answer any SIX questions: (6x2=12)

1. Prove that : $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} (\sin\theta)^{2m-1}\,(cos\theta)^{2n-1}d\theta$
2. Evaluate : $\int_0^\infty y^3\,e^{-2y^5}dy$
3. Show that $\lim\limits_{z\to 0}\frac{xy^2}{x^2+y^4}$ does not exist
4. Verify whether the function $f(z) = e^z$ is an analytic
5. Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the curve $y = x$
6. Name the type of the Singularities for the function $f(z) = \frac{1}{z}\cosh(\frac{1}{z})$
7. Define cross-ratio of 4 points
8. Define Translation. Give an example

SECTION-B

II. Answer any FOUR questions: (4x6=24)

1. a) Show that $p.\beta(p, q+1) = q.\beta(p+1, q)$

b) Evaluate $\int_0^1 \frac{dx}{\sqrt{x\log\left(\frac{1}{x}\right)}}$

2. State and prove the Duplication formula

3. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where m,n>0

4. Show that the locus of a point z satisfying $arg\left(\frac{z-1}{z+1}\right) = \pi/3$ is a circle. Find its Center and Radius

5. State and prove the necessary condition for the function $f(z) = u + iv$ to be analytic

6. Show that the function $u = (r + \frac{1}{r})cos\theta$ is harmonic and find the analytic function.

SECTION-C

III. Answer any FOUR question: **(4x6=24)**

1. If C is a circle with centre ‘a’ and radius ‘r’ then show that

   (i) $\oint \frac{dz}{z-a} = 2\pi i$

   (ii) $\oint (z-a)^n dz = 0$ if $n \neq 0$

2. State and prove Cauchy’s Integral formula

3. Evaluate $\int_c \frac{(z-1)dz}{(z+1)^2(z-2)}$; where $|z - i| = 2$

4. Discuss the transformation $w = sinz$

5. Find the Bilinear transformation which maps $z = -1,\ 0,\ 1\ into\ \ w = 0, i,\ 3i$. Also find the fixed points of the transformation

6. Find the map of the real axis of the z-plane in the w-plane under the transformation $w = \frac{1}{z+i}$. Also find the Invariant point.

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